Preliminary Wavelet Analysis of LongEZ Flights from November 1998

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This report begins by comparing the standard Power Spectral Density (*PSD*) of the FFT with the Corrected Wavelet Mean Energy. Since our wavelets are derived from the FFT, we have taken the magnitudes of each complex set identically as $\sqrt{y_r^2 + y_i^2}$. This gives us non-windowed *PSD* from the FFT coefficients. In the wavelet domain, this magnitude gives us the scalogram. The scalogram is a 2 dimensional entity which gives us normalized energy across time and scale (frequency).

The procedure we use to correct energy from the scalogram to *PSD* is the following:

$$s = \sqrt{y_r^2 + y_i^2}$$
$$PSD = \log(mean(s)) - .5\log(frequency) + \log(1.5)$$

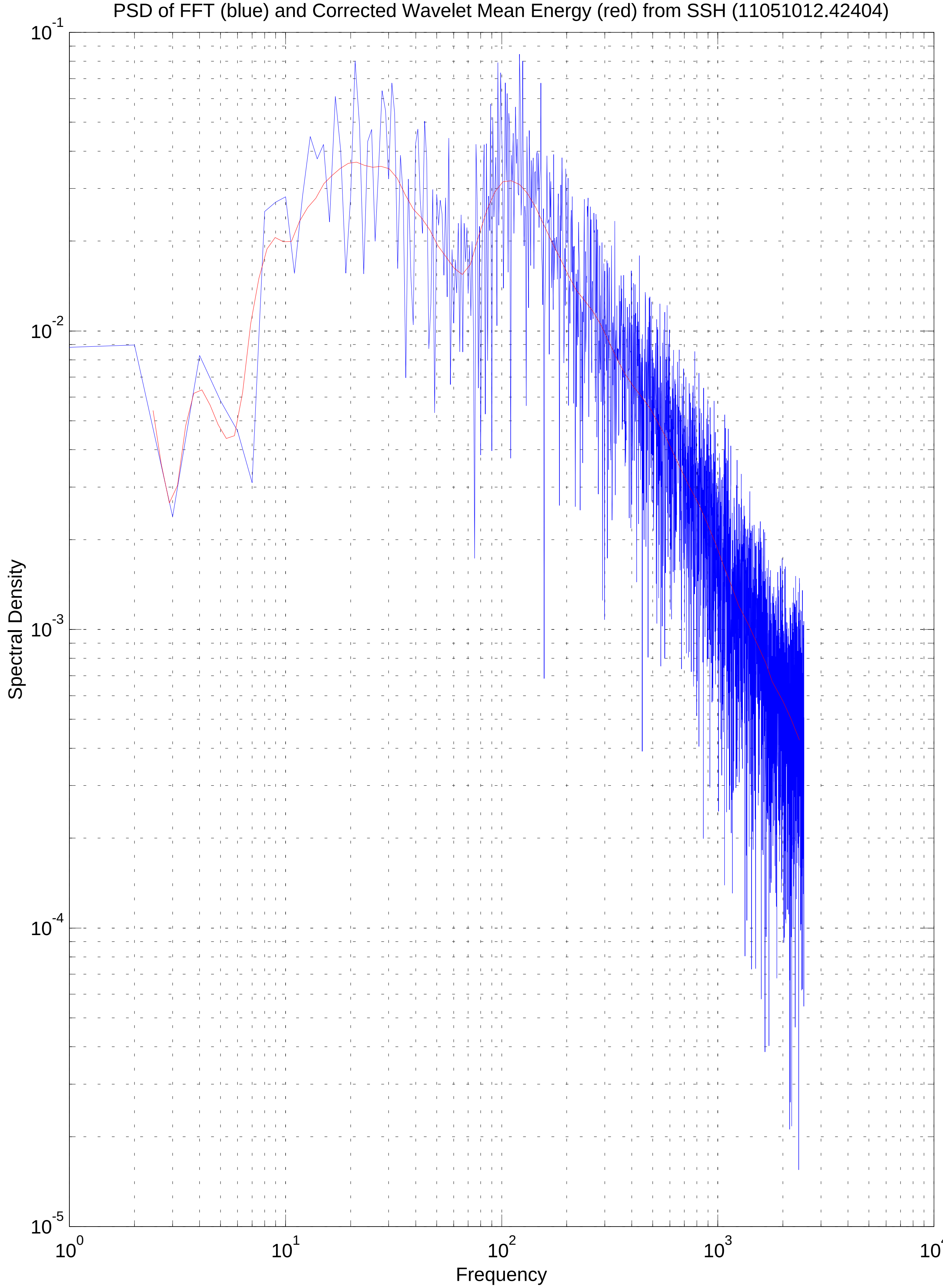
The 1st term in the above *PSD* equation finds the mean value across each wavelet scale. The 2nd term removes the power of 2 energy increase imposed by the Morlet Wavelet. The 3rd term corrects for a global bias shift.

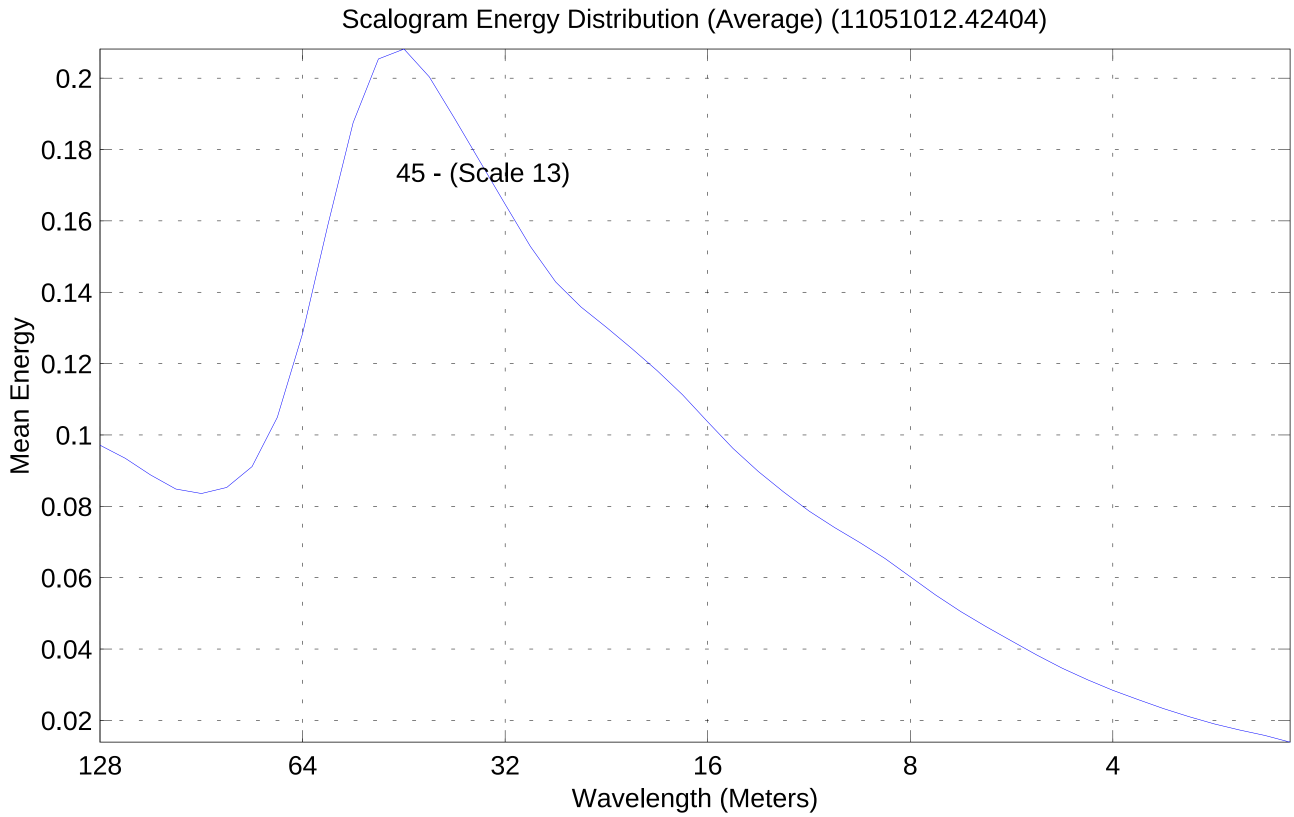
Plots 1 and 4 superimpose *PSD* with Corrected Wavelet Mean Energy. Each plot is from actual LongEZ data.

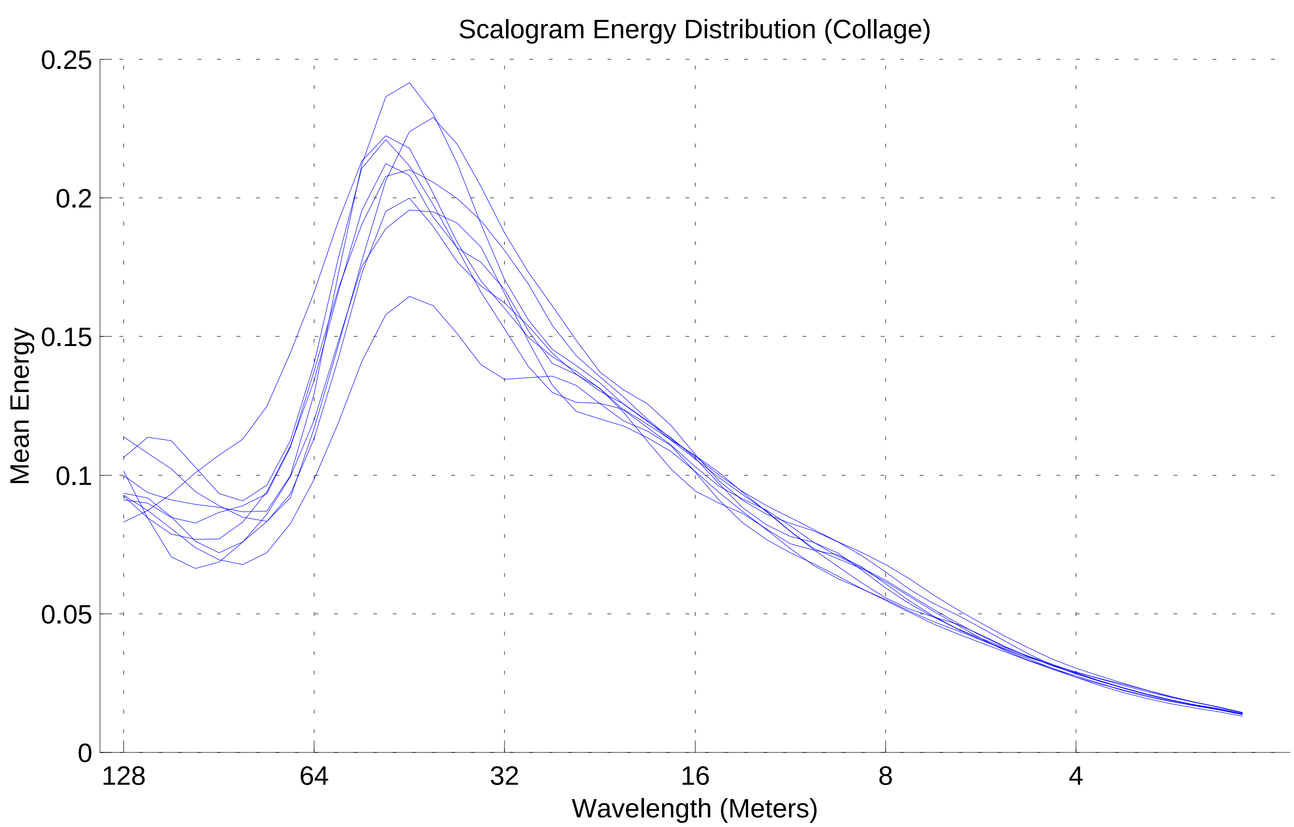
The next desire with this study was to examine longer data files in hopes of achieving a better estimate of phase. To do this, I developed a program that moves through long data files in blocks to determine an average Scalogram Energy Distribution. Plots 2 and 5 are examples of this. Plot 2 represents a flight line where the LongEZ was moving against the direction of the dominant wave producing a wavelength peak at 45 meters. Plot 5 is a flight line in the opposite direction moving with the wave field. The dominant wave now is found at 70 meters with a broadening of energy. This is due to Doppler shift.

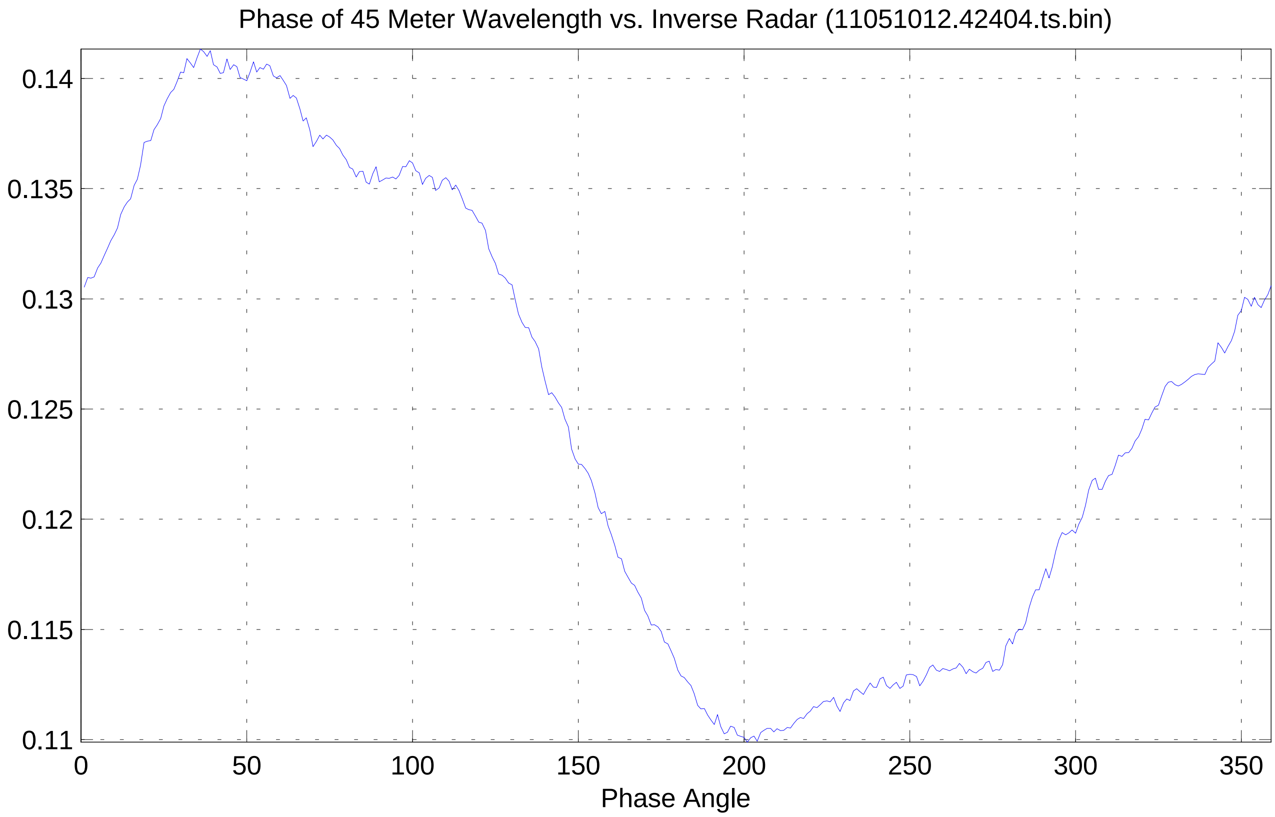
Plots 3 and 6 are SSH phase measurements vs. Radar and Tilt from each flight line. From the previous 2 plots, I used the peak wavelength (scale) from the scalogram to generate phase measurements from the wavelet coefficients. These phase measurements were then applied to both Radar and Tilt. A 51 point running average was finally applied to smooth each data set.

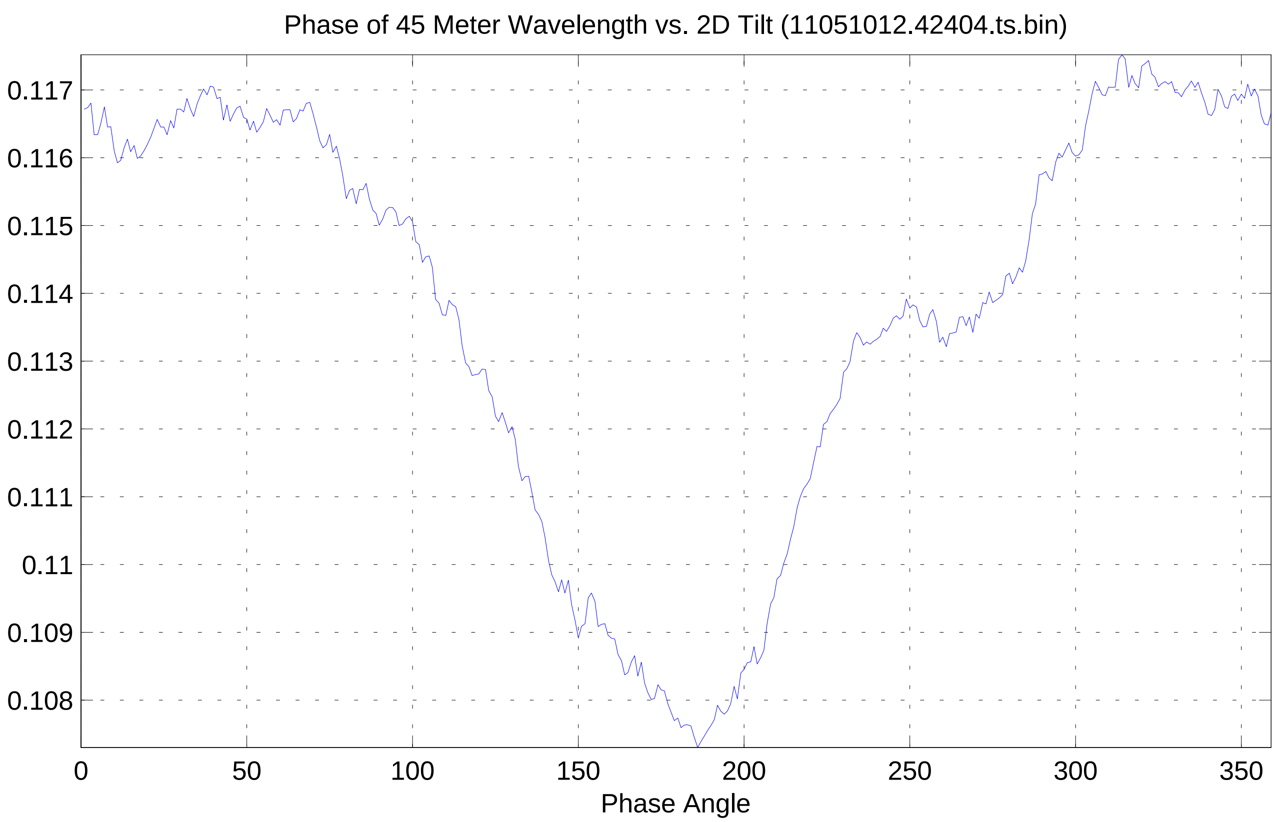
Results are inconclusive at this point. We had hoped the Doppler shift would not effect our phase measurements. Phase vs. Radar for each flight line is similar but Phase vs. Tilt is not. It may be that our flight line with the 70 meter dominant wave is providing 25 more measurements per wave (samples are 1 meter apart) and thus giving us a more accurate phase.

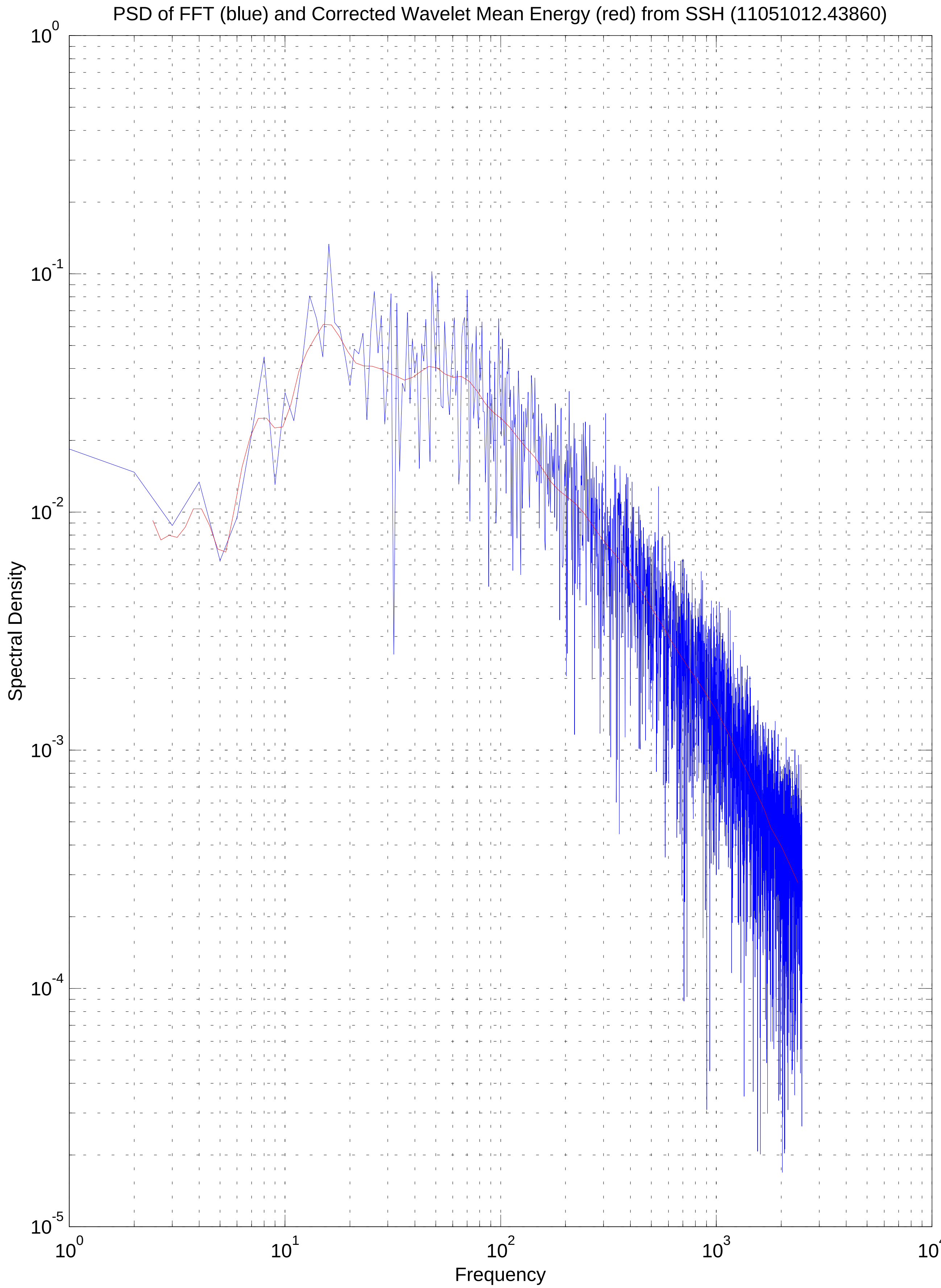

PSD of FFT (blue) and Corrected Wavelet Mean Energy (red) from SSH (11051012.42404)
Spectral Density
Frequency
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}

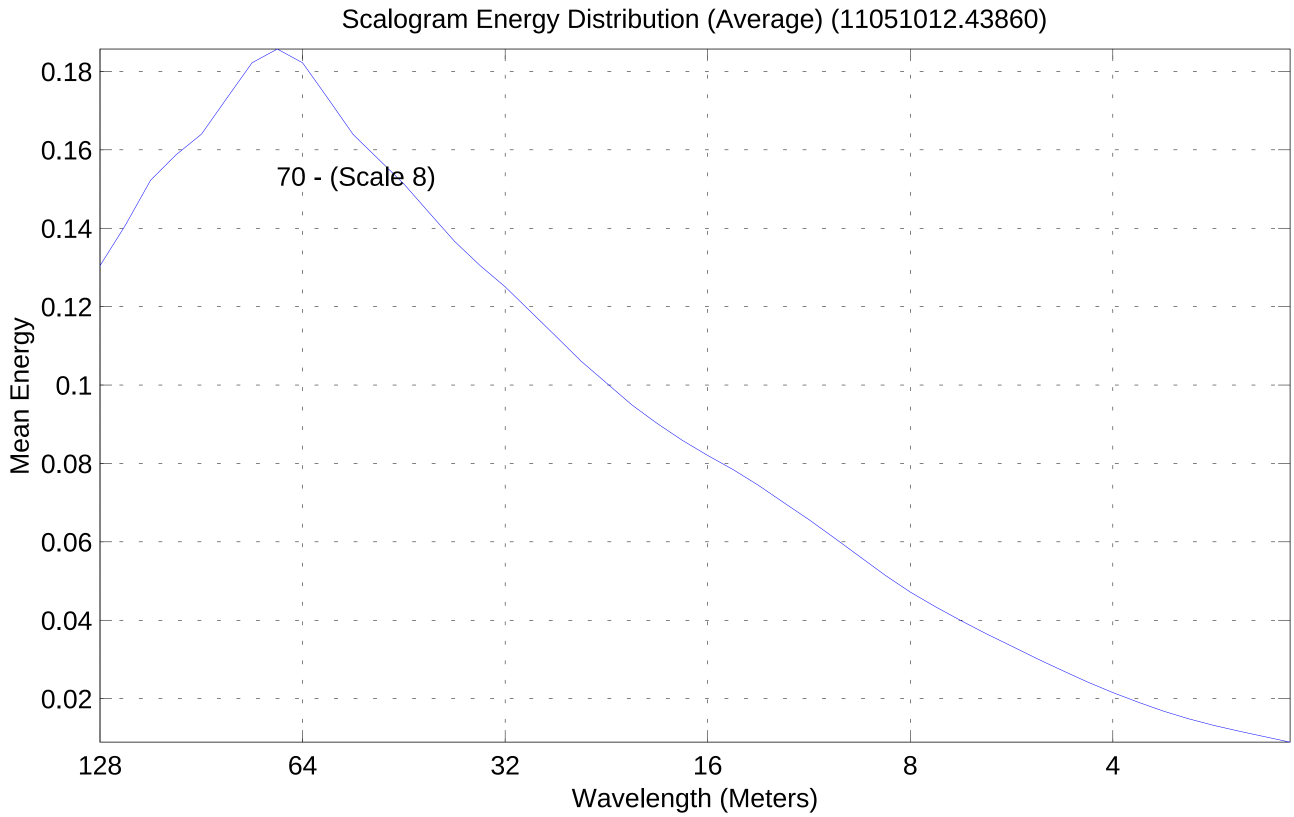

Scalogram Energy Distribution (Average) (11051012.42404)
45 - (Scale 13)
Mean Energy
Wavelength (Meters)
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
128
64
32
16
8
4

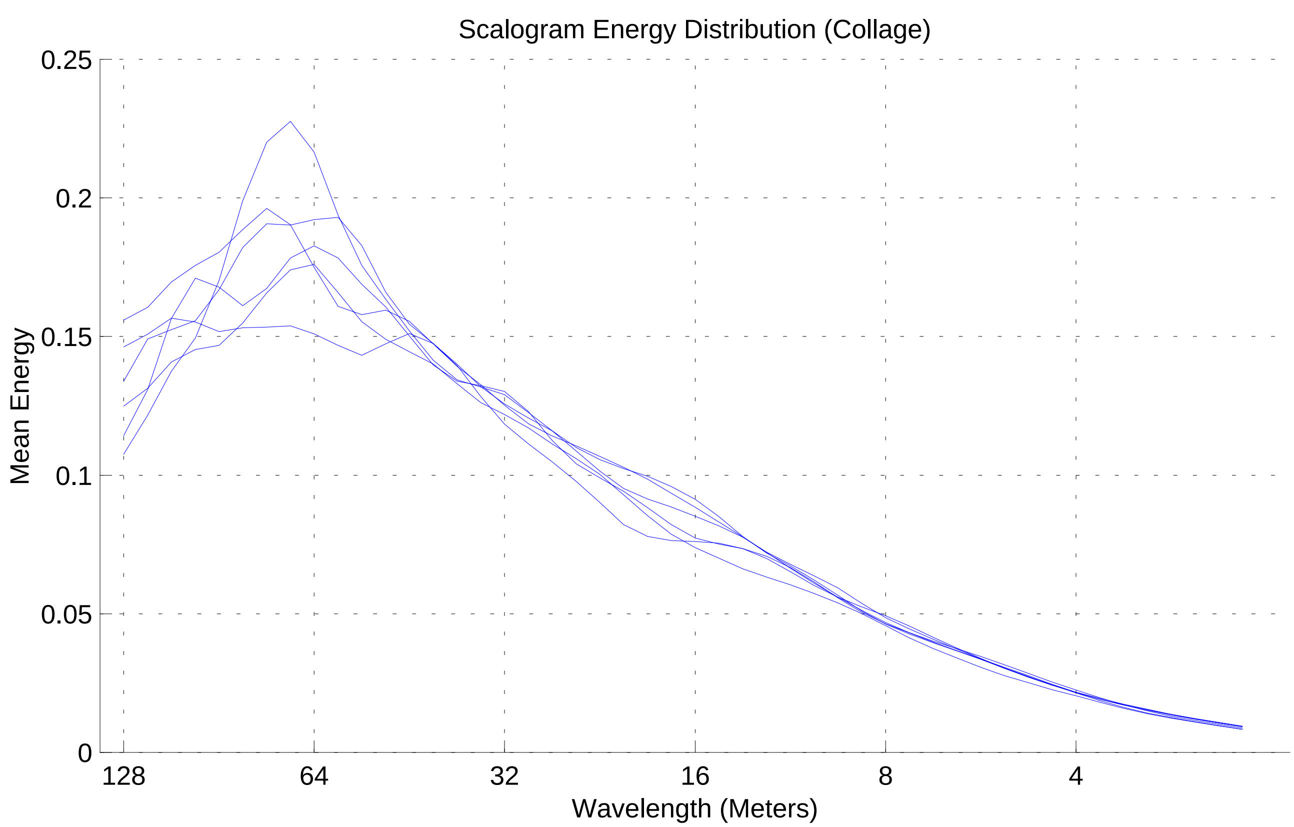

Scalogram Energy Distribution (Collage)
Mean Energy
Wavelength (Meters)
0.25
0.2
0.15
0.1
0.05
0
128
64
32
16
8
4

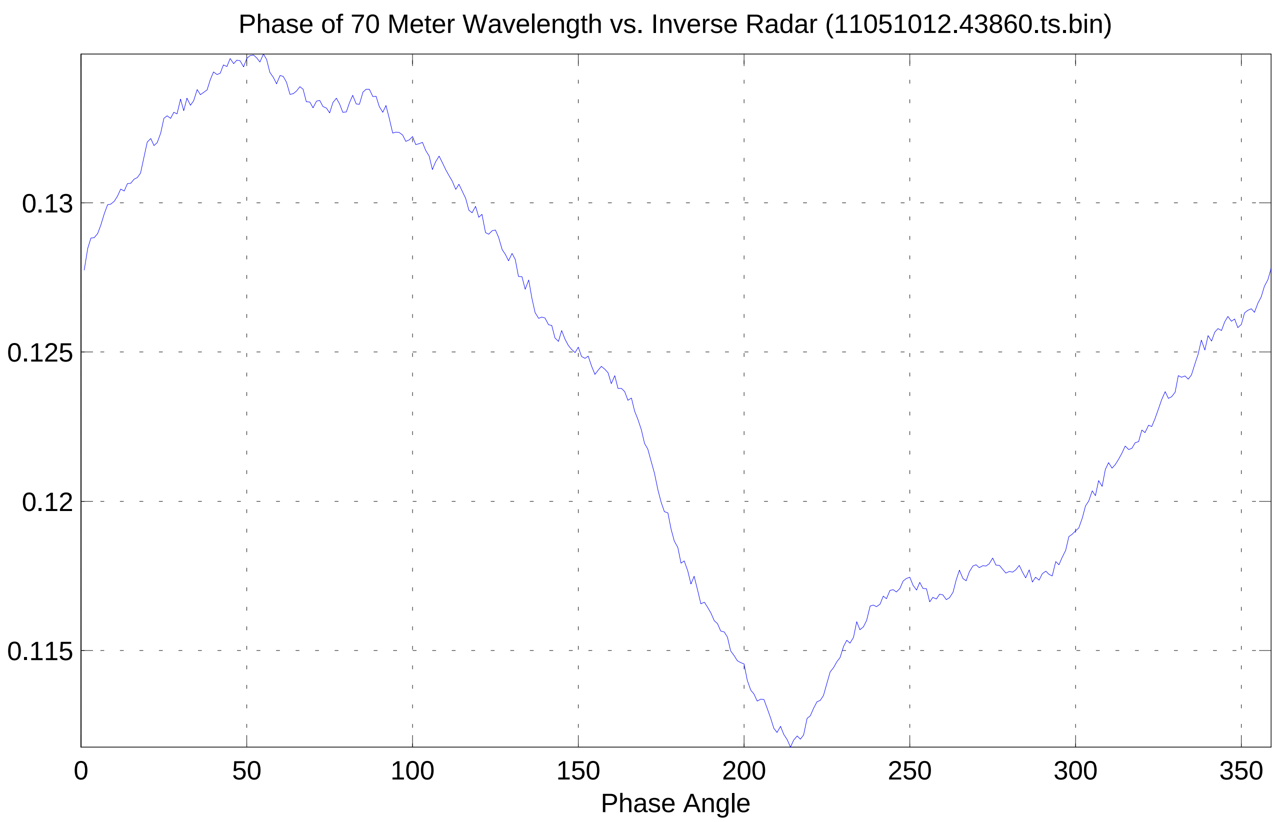

Phase of 45 Meter Wavelength vs. Inverse Radar (11051012.42404.ts.bin)
0.14
0.135
0.13
0.125
0.12
0.115
0.11
0
50
100
150
200
250
300
350
Phase Angle

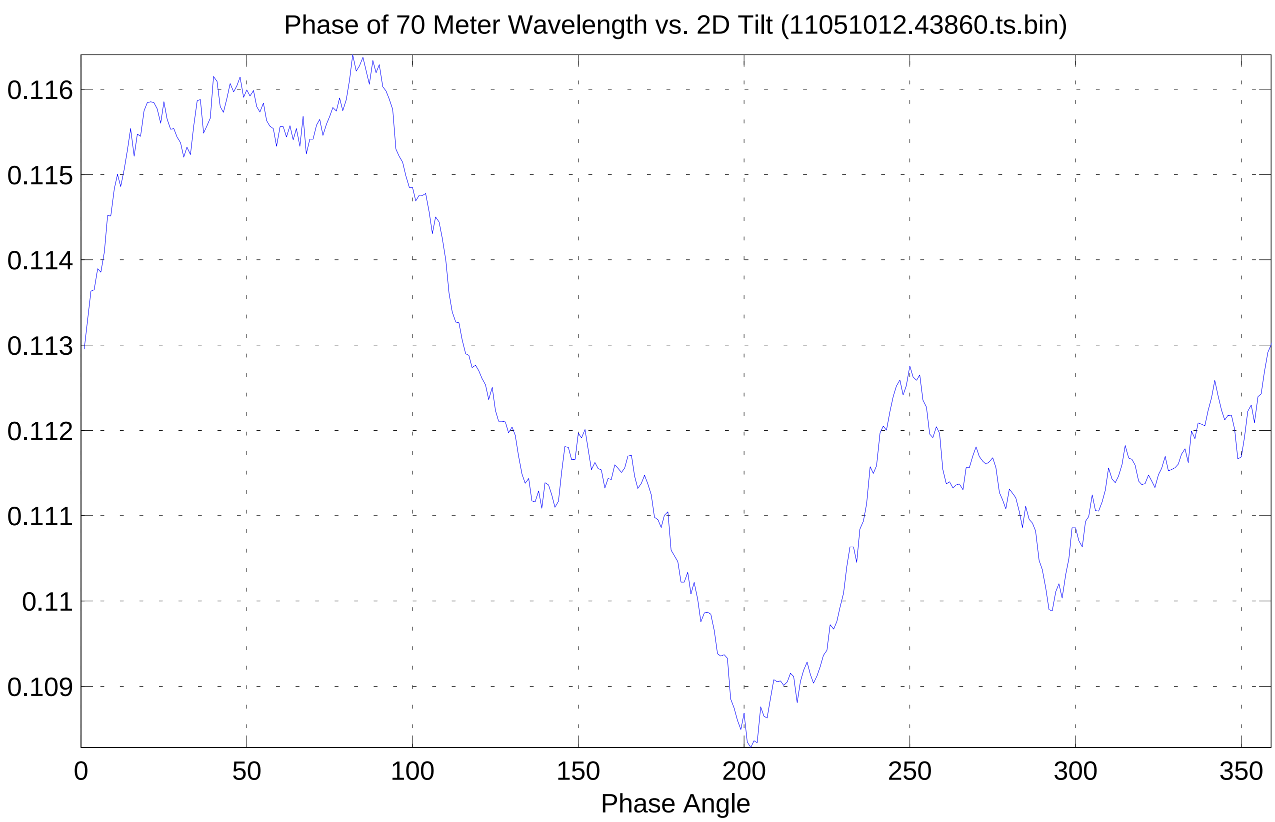

Phase of 45 Meter Wavelength vs. 2D Tilt (11051012.42404.ts.bin)
0.117
0.116
0.115
0.114
0.113
0.112
0.111
0.11
0.109
0.108
0
50
100
150
200
250
300
350
Phase Angle


PSD of FFT (blue) and Corrected Wavelet Mean Energy (red) from SSH (11051012.43860)
Spectral Density
Frequency
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}


Scalogram Energy Distribution (Average) (11051012.43860)
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
Mean Energy
70 - (Scale 8)
128
64
32
16
8
4
Wavelength (Meters)


Scalogram Energy Distribution (Collage)
0.25
0.2
0.15
0.1
0.05
0
Mean Energy
128
64
32
16
8
4
Wavelength (Meters)


Phase of 70 Meter Wavelength vs. Inverse Radar (11051012.43860.ts.bin)
0.13
0.125
0.12
0.115
0
50
100
150
200
250
300
350
Phase Angle


Phase of 70 Meter Wavelength vs. 2D Tilt (11051012.43860.ts.bin)
0.116
0.115
0.114
0.113
0.112
0.111
0.11
0.109
0
50
100
150
200
250
300
350
Phase Angle